Time Value of Money

Financial Management

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Chapter Overview

1. Introduction

The time value of money is the single most important concept in finance. This chapter explores the elements that comprise it, including future value, present value, compounding, and discounting.

2. Key Learning Outcomes

By the end of this, you will be able to:

1. Calculate the Present Value (PV) and Future Value (FV) of a single sum using the Time Value of Money (TVM) principles.
2. Evaluate the impact of varying interest rates and the number of periods on the compounded or discounted value using Time Value of Money (TVM) principles.
3. Explain compounding and discounting.

Introduction

> *Money in the future is less valuable than money today.*

Time Value of Money (TVM) is arguably the single most important concept in finance, and it can be applied to both business and personal activities. Why is money worth more today than the same amount in the future? How can we estimate what today's investment will be worth in the future? How can we decide how much we need to save today for some future need?

This chapter will answer these questions and more. It is split into 6 sections, starting with the time value of money concepts, followed by examples of how to run important calculations for the individual elements in both Excel and on a calculator:

1. Time Value of Money Concepts
2. Calculating Present Value (PV)
3. Calculating Future Value (FV)
4. Calculating Investment Returns (i)
5. Calculating Number of Periods (N)
6. Learning Tools

Future Value

1. Introduction to Future Value

The **Future Value (FV)** equals the value of money invested today at a given interest rate some number of periods (years, months, etc.) in the future.

Working with the TVM principle, we know that future money is worth less to us than the same amount of money today. We'd much rather prefer to have $10 today, than the same $10 in 1 year's time (unless we are acting in what we call *irrational* in investment circles).

Why would a rational investor prefer to have $10 today rather than the same $10 in a year's time? There are several reasons:

1. **Inflation:** This drives the price of goods and services up. $10 in a year's time cannot buy us as much as $10 today.
2. **Earnings:** An investor with $10 today can invest that money and earn a return on it so that in a year's time, it will be more than $10.
3. **Opportunity Cost:** The owner of $10 can choose to enjoy the money now. Why wait? Wouldn't we rather spend that money now rather than have to wait a year? We would earn a psychic benefit from it that could accrue over time.
4. **Risk:** An investor with $10 now already has it. Conversely, an investor with the promise of receiving $10 in a year's time has the risk that the counterparty will not give it to them.

Given the above, a rational investor must have some amount of *expected* money in the future that would convince them to give up $10 today. Would they want $15 in 1 year to give up our $10 today? $20 in 1 year? $30? This principle is the essence of investing. An investor gives up money today (i.e., invests it into a business or an asset) in return for receiving a higher amount in the future. The amount of money that we want in the future to invest our $10 ($1,000 or $1,000,000 or some other amount) today is a function of risk. The greater the risk, the larger the rate at which our money should be expected to grow.

Importantly, we use the word ***expected*** to grow. When we take risks, sometimes it works in our favor, and sometimes it goes against us. Some of those potential risks would be:

- The length of time we have to wait - the longer we have to wait, the more uncertainty, thus the more return we expect to make.
- The risk we perceive in the investment we are making - the higher the risk, the more return we expect to make. Remember, sometimes risky investments lose money, we have to expect enough gain to be willing to risk a loss.

- The risk/return we would receive on other types of investments. The baseline we usually measure against is called the risk-free rate of return, which is usually U.S. Treasuries from U.S. investors.

2. Formula for Future Value

$$\mathrm{FV} = \mathrm{PV} \times (1 + i)^n$$

where:

- **i** is an interest or discount rate
- **n** is the number of periods

Present Value

1. Introduction to Present Value

The Present Value (PV) equals the current or present value of future cash flows, discounted at a selected interest (discount) rate.

This looks at the $10 investment situation from the other direction. If we expect to receive $10 in one year's time, a rational investor would pay less than $10 for that opportunity today.

Again, the amount we would pay today is a function of the risk of the deal, the length of time we have to wait, and the return we can get on other types of investments. Along with other potential risks, these factor into the discount rate. So how much is that $10 in 1 year worth today? That value is known as the Present Value.

2. Discount Rate

If we expect to earn that $10 in the future, we need a number to calculate its present value. The rate we use to estimate how much less that $10 is worth today is called the discount rate. The discount rate is a function of the risk (how likely are we to receive that expected money)? ***Discounting*** means the process of finding the Present Value by reducing future cash flows using that discount rate. Simply, the discount rate is the rate at which our Future Value is reduced to the Present Value.

3. Formula for Present Value

$$\mathrm{PV} = \frac{\mathrm{FV}}{(1+i)^n}$$

where:

- **i** is an interest rate, or discount rate
- **n** is the number of periods

What is Compounding?

Compounding is the process where money earns interest on the initial balance *plus* all previously earned interest. Or, we earn interest on interest. It results in an exponential growth curve.

If we invest $100 in a bank account that generates a 5% interest rate, at the end of the first year, we would have $105. However, how much would you have at the end of the second year?

If you apply simple interest (i.e., non-compounding), where interest is only earned on the principal you invest, we would have $110.00 ($105 + $100 * 0.05) at the end of year 2. However, if we applied compounding, we would have $110.25 (105*1.05). The reason we would have more is that we are also earning interest on the extra $5 of interest we earned at the end of year 1. In a way, the interest we earned in the past becomes principal. While $0.25 might not seem much, another way to think about is an extra 0.25% on our original investment. Had we invested a million, we would have $1,102,500, an extra $2,500.

In real life, most investments and savings follow the law of compound interest, not simple interest. This means we earn interest on our earnings as well as on the initial amount we invest.

This makes a significant difference over time. Simple interest (also known as simple growth) is linear; it increases in a straight line, whereas compounding is exponential.

The further out in time and the higher the interest rate, the greater the compounding effect.

	Periods						
Rate per Period		1	3	5	10	20	50
	4.00%	$104	$112	$122	$148	$219	$711
	8.00%	$108	$126	$147	$216	$466	$4,690
	12.00%	$112	$140	$176	$311	$965	$28,900
	16.00%	$116	$156	$210	$441	$1,946	$167,070
	20.00%	$120	$173	$249	$619	$3,834	$910,044

What is Discounting?

Discounting means the process of finding present value by reducing future cash flows using a discount (interest) rate. Discounting is the inverse of compounding.

As opposed to an interest rate, it is called a discount rate since it reduces future values back to their present values. The discount rate reflects risk and expected return.

A higher risk expectation leads to a higher discount rate because there is less certainty of receiving those future cash flows. The present value of a future sum decreases more rapidly with higher discount rates and longer time periods.

Calculating the Present Value (Excel)

1. The Present Value Formula

To calculate the present value in MS Excel, use the following formula:

$$\mathrm{PV} = \mathrm{PV}(\mathrm{rate}, \mathrm{nper}, \mathrm{pmt}, [\mathrm{fv}], [\mathrm{type}])$$

- Rate is the interest rate.
- NPER is the number of periods.
- PMT is a recurring payment stream.
- FV is the expected value of a certain dollar amount N periods in the future.
- TYPE (optional) is whether the cash flow is an ordinary (regular) annuity where payments are made at the end of a period (0, or blank/default) or an annuity due where payments are made at the beginning of a period (1).

2. Example Calculation

- Goal: $5,000 for a trip in 3 years
- Assumed investment return: 8.00%

Time Value of Money Inputs:

1. N (Number of periods): 3 years
2. i (Interest rate): 8.00%
3. PV (Present Value): To be calculated
4. PMT (Payment): $0 (not used in this example)
5. FV (Future Value): $5,000

$$\mathrm{PV} = \mathrm{PV}(\mathrm{rate}, \mathrm{nper}, \mathrm{pmt}, [\mathrm{fv}], [\mathrm{type}])$$

$$\mathrm{PV} = \mathrm{PV}(8.00\%, 3, 0, 5000)$$

$$\mathrm{PV} = \$3{,}969.16$$

Calculating the Present Value (Calculator)

1. Calculating PV with BA II Plus

1. Enter the Future Value (FV):
    - Key in **5000**.
    - Press the **FV** key.
2. Enter the Interest Rate (i):
    - Key in **8.00**.
    - Press the **I/YR** button.
3. Enter the Number of Periods (n):
    - Key in **3**.
    - Press the **N** button.
4. Enter the Periodic Payment (PMT):
    - Key in **0**.
    - Press the **PMT** button.
5. Calculate the Present Value:
    - Press the **CPT** button.
    - Press the **PV** button. The calculator will now display the Present Value.

2. Calculating PV with HP10bII+

1. Enter the Future Value (FV):
    - Key in **5000**.
    - Press the **FV** key.
2. Enter the Interest Rate (i):
    - Key in **8.00**.

- Press the **I/YR** button.

3. Enter the Number of Periods (n):
    - Key in **3**.
    - Press the **N** button.
4. Enter the Periodic Payment (PMT):
    - Key in **0**.
    - Press the **PMT** button.
5. Calculate the Present Value:
    - Press the **PV** button. The calculator will now display the Present Value.

After following these steps on the HP10bII+, the calculator should display a present value of $3,969.16, which matches the result in the video.

***Note**: If you are dealing with periodic compounding (like monthly or quarterly), ensure that you adjust the **i** (interest rate) and **n** (number of periods) accordingly. For instance, if compounding is monthly, and you are considering a 5-year period, **n** will be 60 (12 months x 5 years). Similarly, if the annual interest rate is 5.00% and it is compounded monthly, then the monthly interest rate will be 5.00/12.*

Calculating the Future Value (Excel)

1. The Future Value Formula

To calculate the future value in MS Excel, use the following formula:

$$\mathrm{FV} = \mathrm{FV}(\mathrm{rate}, \mathrm{nper}, \mathrm{pmt}, [\mathrm{pv}], [\mathrm{type}])$$

- Rate is the interest rate.
- NPER is the number of periods.
- PMT is a recurring payment stream.
- PV is the present value of current monetary holdings.
- TYPE (optional) is whether the cash flow is an ordinary (regular) annuity where payments are made at the end of a period (0, or blank/default) or an annuity due where payments are made at the beginning of a period (1).

2. Example Calculation

- Initial Investment: $24 in 1626
- Time Period: 400 years (assuming calculation to 2026)
- Assumed Annual Return: 10.00%

Time Value of Money Inputs:

1. PV = -$24 (negative, as it represents an outflow)
2. N = 400 years
3. I = 10.00%
4. PMT = 0
5. FV = To be calculated

$$\mathrm{FV} = \mathrm{FV}(\mathrm{rate}, \mathrm{nper}, \mathrm{pmt}, [\mathrm{pv}], [\mathrm{type}])$$

$$\mathrm{FV} = \mathrm{FV}(10.00\%, 400, 0, -24)$$

$$\text{FV} = \$865 \text{ quadrillion}$$

$$\text{FV} = \$865{,}536{,}336{,}660{,}610{,}000.00$$

Calculating the Future Value (Calculator)

1. Calculating FV with BA II Plus

1. **Enter the Present Value (PV)**:
    - Key in **24**.
    - Press the **PV** key.
2. **Key in the Interest Rate (i)**:
    - Input **10.00** (representing the 10% interest rate per period).
    - Press the **I/YR** button.
3. **Specify the Number of Periods (n)**:
    - Key in **400** (representing 400 periods).
    - Press the **N** button.
4. **Enter the Periodic Payment (PMT)**:
    - Key in **0**.
    - Press the **PMT** button.
5. **Calculate the Future Value**:
    - Press the **CPT** button.
    - Press the **FV** button.

2. Calculating FV with HP10bII+

1. **Enter the Present Value (PV)**:
    - Key in **24**.
    - Press the **PV** key.
2. **Key in the Interest Rate (i)**:
    - Input **10.00** (representing the 10% interest rate per period).

- Press the **I/YR** button.

3. **Specify the Number of Periods (n)**:
    - Key in **400** (representing 400 periods).
    - Press the **N** button.
4. **Enter the Periodic Payment (PMT)**:
    - Key in **0**.
    - Press the **PMT** button.
5. **Calculate the Future Value**:
    - Now, press the **FV** button.

***Note**: If you are dealing with periodic compounding (like monthly or quarterly), ensure that you adjust the **i** (interest rate) and **n** (number of periods) accordingly. For instance, if compounding is monthly, and you are considering a 5-year period, **n** will be 60 (12 months x 5 years). Similarly, if the annual interest rate is 5.00% and it is compounded monthly, then it will be 5.00/12.*

Calculating Return on Investment (Excel)

1. The ROI Formula

To calculate the return on investment in MS Excel, use the following formula:

$$\mathrm{i} = \mathrm{RATE}(\mathrm{nper}, \mathrm{pmt}, \mathrm{pv}, \mathrm{fv}, [\mathrm{type}], [\mathrm{guess}])$$

- Rate is the interest rate.
- NPER is the number of periods.
- PMT is a recurring payment stream.
- PV is the current value of current monetary holdings.
- *TYPE (optional) is whether the cash flow is ordinary (regular) annuity where payments are made: at the end of a period (0, or blank/default) or an annuity due where payment are made at the beginning of a period (1).*
- *Guess (optional) is where the user inputs a guess as to the answer. Excel solves this problem iteratively and may, occasionally, need a starting point closer to the ultimate answer to properly solve a given set of inputs.*

2. Example Calculation

- Initial Investment: $1,000
- Future Value: $1,500
- Time Period: 5 years

Time Value of Money Inputs:

1. N = 5 years
2. PV = -$1,000 (negative because it represents an outflow)
3. PMT = 0
4. FV = $1,500
5. i = To be calculated

$$\mathrm{I} = \mathrm{RATE}(\mathrm{nper}, \mathrm{pmt}, \mathrm{pv}, \mathrm{fv}, [\mathrm{type}], [\mathrm{guess}])$$

$$I = \text{RATE}(5, 0, -1000, 1500)$$

$$I = 8.45\%$$

Calculating Return on Investment (Calculator)

1. Calculating ROI (rate) with BA II Plus

1. **Enter the Present Value (PV)**:
    - Input **-1000** (Note: The present value should be entered as a negative since it is an outflow of cash).
    - Press the **PV** key.
2. **Enter the Future Value (FV)**:
    - Input **1500**.
    - Press the **FV** key.
3. **Enter the Number of Periods (n)**:
    - Key in **5**.
    - Press the **N** button.
4. **Enter the Periodic Payment (PMT)**:
    - Key in **0** since there is no periodic payment.
    - Press the **PMT** button.
5. **Calculate the Interest Rate**:
    - Press the **CPT** button.
    - Press the **I/YR** button.

2. Calculating ROI (rate) with HP10bII+

1. **Enter the Present Value (PV)**:
    - Input **-1000** (Note: The present value should be entered as a negative since it is an outflow of cash).
    - Press the **PV** key.
2. **Enter the Future Value (FV)**:

- Input **1500**.
- Press the **FV** key.

3. **Enter the Number of Periods (n)**:
    - Key in **5**.
    - Press the **N** button.
4. **Enter the Periodic Payment (PMT)**:
    - Key in **0** since there's no periodic payment.
    - Press the **PMT** button.
5. **Calculate the Interest Rate**:
    - Press the **I/YR** button.

***Note**: If you are dealing with periodic compounding (like monthly or quarterly), ensure that you adjust the **i** (interest rate) and **n** (number of periods) accordingly. For instance, if compounding is monthly, and you are considering a 5-year period, **n** will be 60 (12 months x 5 years). Similarly, if the annual interest rate is 5.00% and it is compounded monthly, then it will be 5.00/12.*

Dream Stereo

1. Scenario

- Goal: Leon wants to purchase a home stereo system
- Total Cost: $4,000
- Current Savings: $3,000
- Shortfall: $1,000

Leon cannot contribute additional funds to his savings and is seeking alternative methods to reach his financial goal.

Proposed Solution:

- An investment opportunity with an 8.00% annual return.

2. Question

Determine the time required for the current savings ($3,000) to reach the target amount ($4,000) when invested at an 8.00% annual return.

This scenario demonstrates the practical application of time value of money concepts in personal finance. It requires calculating the number of periods (N) given the present value (PV), future value (FV), and interest rate (i), with no additional payments (PMT). In the following two videos, you will learn how to solve for the number of periods using Excel and a calculator.

Calculating the Periods (Excel)

1. The Number of Periods Formula

To calculate the number of periods in Microsoft Excel, use the following formula:

$$N = \text{NPER}(\text{rate}, \text{pmt}, \text{pv}, \text{fv}, [\text{type}])$$

- Rate is the interest rate.
- NPER is the number of periods.
- PMT is a recurring payment stream.
- PV is the present value of current monetary holdings.
- *TYPE (optional) is whether the cash flow is an ordinary (regular) annuity where payments are made at the end of a period (0, or blank/default) or an annuity due where payments are made at the beginning of a period (1).*
- *Guess (optional) is where the user inputs a guess as to the answer. Excel solves this problem iteratively and may, occasionally, need a starting point closer to the ultimate answer to properly solve a given set of inputs.*

2. Example Calculation

- Current savings: $3,000
- Target amount: $4,000
- Expected annual return: 6.00%

Time Value of Money Inputs:

1. N = To be calculated
2. i = 6.00%
3. PV = -$3,000 (negative because it represents an amount already saved)
4. PMT = 0
5. FV = $4,000

$$N = \text{NPER}(\text{rate}, \text{pmt}, \text{pv}, \text{fv}, [\text{type}])$$

$$N = \text{NPER}(6.00\%, 0, -3000, 4000)$$

$$N = 4.94 \text{ years}$$

Calculating Number of Periods (Calculator)

1. Calculating the Number of Periods with BA II Plus

1. **Enter the Present Value (PV):**
    - Key in **-3000** (negative because it is an outflow of money).
    - Press the **PV** key (this will store the present value).
2. **Enter the Future Value (FV):**
    - Key in **4000**.
    - Press the **FV** key (this will store the future value).
3. **Enter the Interest Rate (i):**
    - Key in **6.00** (representing the 6.00% annual interest rate).
    - Press the **I/YR** button (this will store the interest rate per year).
4. **Enter the Periodic Payment (PMT):**
    - Key in **0**.
    - Press the **PMT** button (this will store the periodic payment).
5. **Compute the Number of Periods:**
    - Press the **CPT** button.
    - Press the **N** button.

2. Calculating the Number of Periods with HP10bII+

1. **Enter the Present Value (PV):**
    - Key in **-3000** (negative because it is an outflow of money).
    - Press the **PV** key (this will store the present value).
2. **Enter the Future Value (FV):**
    - Key in **4000**.

- Press the **FV** key (this will store the future value).

3. **Enter the Interest Rate (i):**
    - Key in **6.00** (representing the 6.00% annual interest rate).
    - Press the **I/YR** button (this will store the interest rate per year).
4. **Enter the Periodic Payment (PMT):**
    - Key in **0**.
    - Press the **PMT** button (this will store the periodic payment).
5. **Compute the Number of Periods:**
    - Now, press the **N** button.

Note: *If you are dealing with periodic compounding (like monthly or quarterly), ensure that you adjust the i (interest rate) accordingly. For instance, if compounding is monthly and the annual interest rate is 5.00%, it will be 5.00/12.*

Summary

1. Introduction

In this chapter, we covered the following sections:

1. Time Value of Money Concepts
2. Calculating Present Value (PV)
3. Calculating Future Value (FV)
4. Calculating Investment Returns (i)
5. Calculating Number of Periods (N)

2. Time Value of Money Concepts

Present Value (PV) and **Future Value (FV)** are fundamental concepts in finance. The relationship between PV and FV is governed by the time value of money principle, which states that a dollar today is worth more than a dollar in the future due to its potential earning capacity.

- **Present Value (PV)**: This is the current value of a future sum of money or stream of cash flows, given a specified discount rate. It answers the question, "How much are future dollars worth today?"
- **Future Value (FV)**: This is the value of a current asset at a future date based on an assumed rate of growth. It answers the question, "How much will a current amount be worth in the future?"
- **Compounding** refers to the process of earning interest on both the initial principal and the accumulated interest from previous periods.
- **Discounting** is the process of determining the present value of a future amount. It is essentially the reverse of compounding, where future values are brought back to their present value using a discount rate.

3. Calculating Present Value (PV)

To calculate the present value, use the formula:

$$\mathrm{PV} = \frac{\mathrm{FV}}{(1+i)^n}$$

Where:

- PV = Present Value
- FV = Future Value
- i = Interest rate per period
- N = Number of periods

This formula discounts the future value back to the present by accounting for the interest rate over a number of periods. To calculate the present value using Excel, use the formula:

$$\mathrm{PV} = \mathrm{PV}(\mathrm{rate}, \mathrm{nper}, \mathrm{pmt}, [\mathrm{fv}], [\mathrm{type}])$$

4. Calculating Future Value (FV)

To calculate the future value, use the formula:

$$\mathrm{FV} = \mathrm{PV} \times (1+i)^n$$

This formula compounds the present value forward to the future by applying the interest rate over a number of periods. To calculate the future value using Excel, use the formula:

$$\mathrm{FV} = \mathrm{FV}(\mathrm{rate}, \mathrm{nper}, \mathrm{pmt}, [\mathrm{pv}], [\mathrm{type}])$$

5. Calculating Investment Returns (i)

To calculate the interest rate or investment return in Excel, use the formula:

$$\mathrm{i} = \mathrm{RATE}(\mathrm{nper}, \mathrm{pmt}, \mathrm{pv}, \mathrm{fv}, [\mathrm{type}], [\mathrm{guess}])$$

This formula helps determine the rate of return required to grow the present value to the future value over the specified number of periods.

6. Calculating Number of Periods (N)

To calculate the number of periods in Excel, use the formula:

$$N = \text{NPER}(\text{rate}, \text{pmt}, \text{pv}, \text{fv}, [\text{type}])$$

This formula helps determine how long it will take for the present value to grow to the future value at a given interest rate.